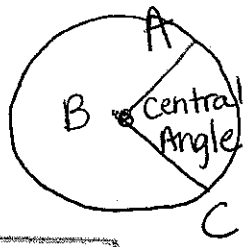
10.2 Measuring Angles and Arcs

Target: Use properties of central angles to find angle and arc measures.

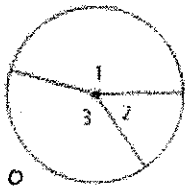
1 Angles and Arcs A central angle of a circle is an angle with a vertex in the center of the circle. Its sides contain two radii of the circle. $\angle ABC$ is a central angle of $\odot B$.



KeyConcept Sum of Central Angles

Words The sum of the measures of the central angles of a circle with no interior points in common is 360.

Example $m\angle 1 + m\angle 2 + m\angle 3 = 360$



All angles of a circle add up to 360°

Example 1 Find Measures of Central Angles

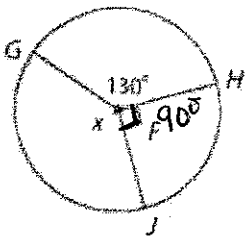
Find the value of x.

$x + 90 + 130 = 360$

$x + 220 = 360$

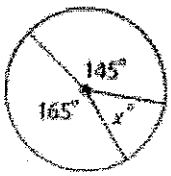
$-220 \quad -220$

$x = 140°$



GuidedPractice

1A.

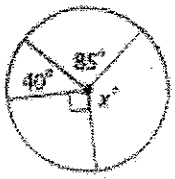


$x + 145 + 165 = 360$

$x + 310 = 360$

$x = 50$

1B.



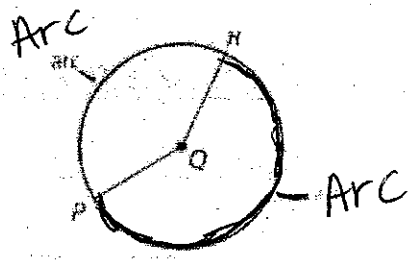
$x + 40 + 85 + 90 = 360$

$x + 215 = 360$

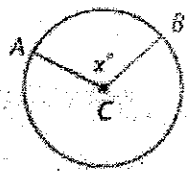
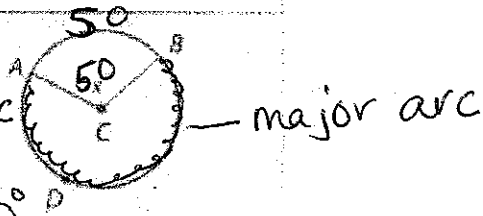
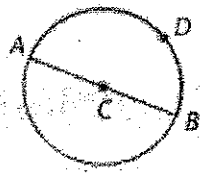
$-215 \quad -215$

$x = 145$

An **arc** is a portion of a circle defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.

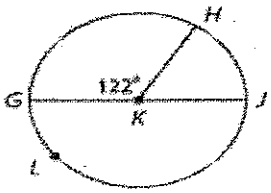


KeyConcept Arcs and Arc Measure

Arc	Measure
A **minor arc** is the shortest arc connecting two endpoints on a circle.	Less than 180° Same measure as the central angle $\widehat{AB} = 50°$ 
A **major arc** is the longest arc connecting two endpoints on a circle.	Greater than 180° Major arc = 360 − minor arc $\widehat{ADB} = 360 - 50$ $\widehat{ADB} = 310°$ 
A **semicircle** is an arc with endpoints that lie on a diameter.	Equal to 180° must be a diameter 

Example 2 Classify Arcs and Find Arc Measures

$\overline{GJ}$ is a diameter of ⊙K. Identify each arc as a *major arc, minor arc,* or *semicircle*. Then find its measure.



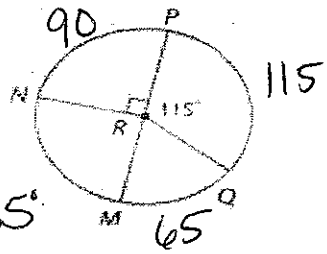
a. $m\widehat{GH}$ = 122°
minor

b. $m\widehat{GLH}$ 360 − 122°
major $m\widehat{GLH} = 238°$

c. $m\widehat{GLJ}$
Semi circle
180°

Guided Practice

$\overline{PM}$ is a diameter of ⊙R. Identify each arc as a *major arc, minor arc,* or *semicircle*. Then find its measure.



2A. $\widehat{MQ}$ 180 − 115 =
minor
65°

2B. $\widehat{MNP}$
Semi circle
180°

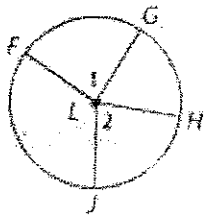
2C. $\widehat{MNQ}$ 360 − 65°
major
295°

Congruent arcs are arcs in the same or congruent circles that have the same measure.

Theorem 10.1

Words In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.

Example If $\angle 1 \cong \angle 2$, then $\widehat{FG} \cong \widehat{HJ}$.
If $\widehat{FG} \cong \widehat{HJ}$, then $\angle 1 \cong \angle 2$.



Real-World Example 3 Find Arc Measures in Circle Graphs

SPORTS Refer to the circle graph. Find each measure.

18% ?

a. $m\widehat{CD}$

minor

.18

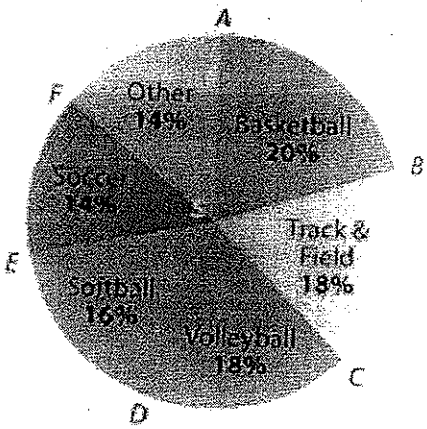
.18 X 360

64.8°

b. $m\widehat{BC}$

64.8°

Female Participation in Sports



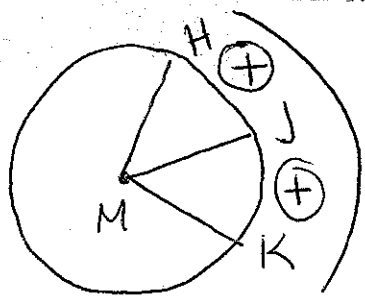
Guided Practice

3A. $m\widehat{EF}$ 14%

minor .14 X 360 = 50.4°

3B. $m\widehat{FA}$ 50.4°

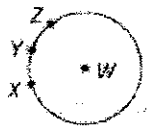
Adjacent arcs are arcs in a circle that have exactly one point in common. In $\odot M$, $\widehat{HJ}$ and $\widehat{JK}$ are adjacent arcs. As with adjacent angles, you can add the measures of adjacent arcs.



Postulate 10.1 Arc Addition Postulate

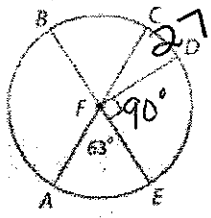
Words The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example $m\widehat{XYZ} = m\widehat{XY} + m\widehat{YZ}$



Example 4 Use Arc Addition to Find Measures of Arcs

Find each measure in ⊙F.



a. $m\widehat{AED}$

$63 + 90 = 153°$

b. $m\widehat{ADB}$

$180° + 63° = 243°$

Guided Practice

4A. $m\widehat{CE}$

$180 - 153 = 27$

$90 + 27 = 117°$

4B. $m\widehat{ABD}$

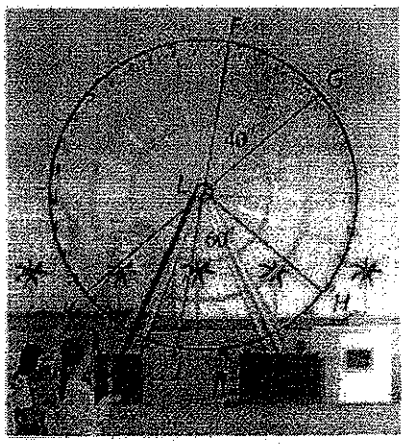
$180 + 27 =$

$207°$

Why is this important?

- Amusement Parks

ENTERTAINMENT Use the Ferris wheel shown to find each measure.



$m\widehat{FG}$

40

$m\widehat{JH}$

60

$m\widehat{KF}$

180

$m\widehat{FH}$

$360 - 60 = 300$