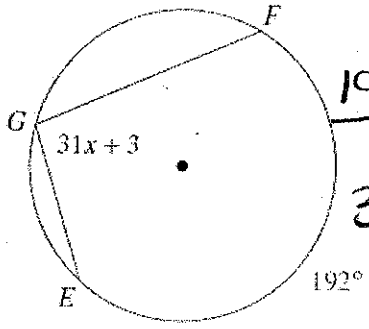
April 7, 2014

#2+#3 Opposite Angles are Supp. = 180°

10.6 Warm-Up

Solve for each variable

1. Inscribed Angle $= \frac{\text{Arc}}{2}$



$\frac{192}{2} = 96$

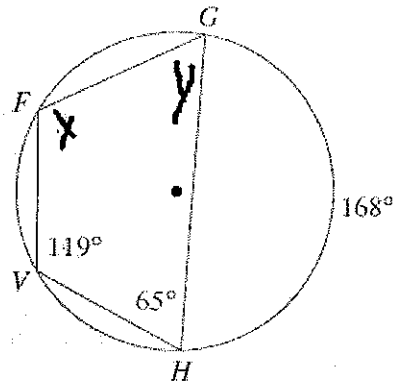
$31x + 3 = 96$

$-3 \quad -3$

$31x = 93$

$\boxed{x = 3}$

2.



$x + 65 = 180$

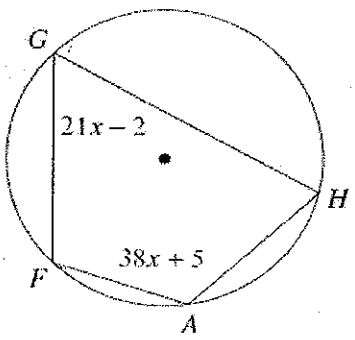
$\boxed{x = 115}$

$y + 119 = 180$

$\boxed{y = 61}$

3.

Find $m\overset{\frown}{FGH}$



$21x - 2 + 38x + 5 = 180$

$59x + 3 = 180$

$59x = 177$

$\boxed{x = 3}$

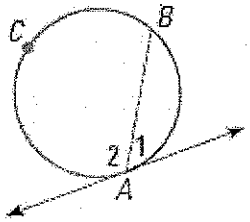
10.6 Secants, Tangents, and Angle Measures

Target: Use properties of secants, tangents and angles to solve problems.

THEOREM

THEOREM 10.12 ★Inscribed Angles

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



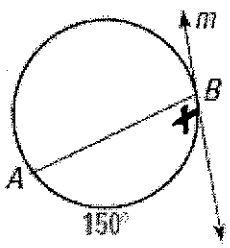
$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$ $\quad m\angle 1 = \frac{\overset{\frown}{AB}}{2}$

$m\angle 2 = \frac{1}{2} m\overset{\frown}{ACB}$ $\quad m\angle 2 = \frac{\overset{\frown}{ACB}}{2}$

EXAMPLE 1 *Finding Angle and Arc Measures*

Line m is tangent to the circle. Find the measure of the red angle or arc.

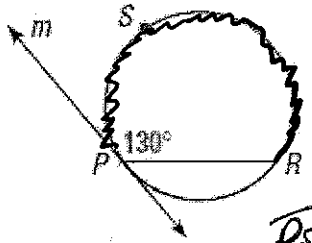
a.



$\frac{150}{2} =$

$m\angle X = \boxed{75°}$

b.



$m\overset{\frown}{PSR} = ?$

$130 \cdot 2 =$

$\overset{\frown}{PSR} = \boxed{260°}$

EXAMPLE 2 *Finding an Angle Measure*

In the diagram below, $\overleftrightarrow{BC}$ is tangent to the circle. Find $m\angle CBD$.

2Angle = Arc

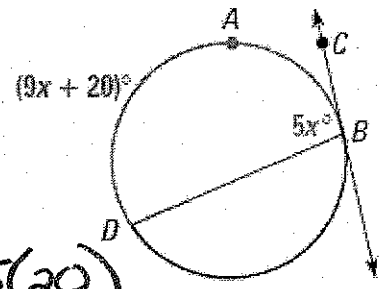
$2(5x) = 9x + 20$

$10x = 9x + 20$

$-9x \quad -9x$

$1x = 20$

$x = \boxed{20}$

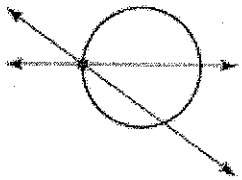


$\angle CBD = 5(20)$

$\boxed{\angle CBD = 100}$

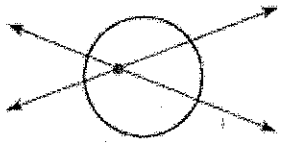
GOAL 2 LINES INTERSECTING INSIDE OR OUTSIDE A CIRCLE

If two lines intersect a circle, there are three places where the lines can intersect.



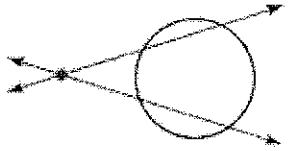
on the circle

Inscribed



inside the circle

Add



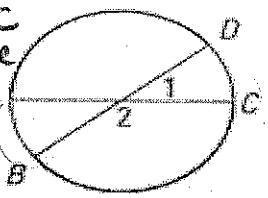
outside the circle

Subtract

You know how to find angle and arc measures when lines intersect *on* the circle. You can use Theorems 10.13 and 10.14 to find measures when the lines intersect *inside* or *outside* the circle. You will prove these theorems in Exercises 40 and 41.

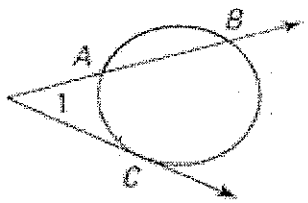
Inside the Circle

Use arc the angle is eating



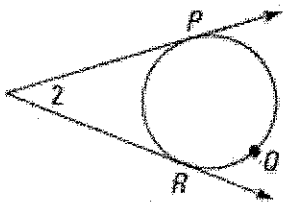
$\angle 1 = \frac{\overset{\frown}{DC} + \overset{\frown}{AB}}{2}$ $\angle 2 = \frac{\overset{\frown}{AD} + \overset{\frown}{BC}}{2}$

Outside the Circle 1



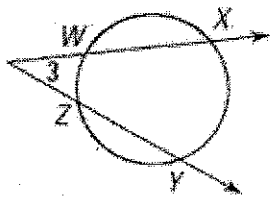
$\angle 1 = \frac{\overset{\frown}{BC} - \overset{\frown}{AC}}{2}$

Outside the Circle 2



$\angle 2 = \frac{\overset{\frown}{PQR} - \overset{\frown}{PQ}}{2}$

Outside the Circle 3



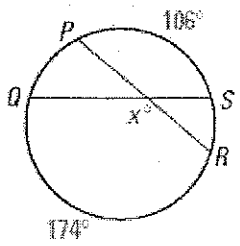
$\angle 3 = \frac{\overset{\frown}{XY} - \overset{\frown}{WZ}}{2}$

EXAMPLE 3 *Finding the Measure of an Angle Formed by Two Chords*

Find the value of x. Inside → Add

$x = \frac{\overset{\frown}{QR} + \overset{\frown}{PS}}{2} = \frac{174 + 106}{2} = \frac{280}{2}$

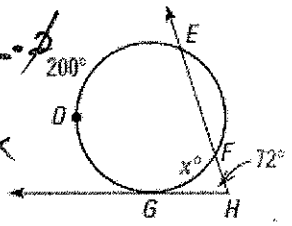
$x = 140$



EXAMPLE 4 *Using Theorem 10.14*

Find the value of x. Outside → Subtract

a.



$2 \cdot 72 = \frac{200 - x}{2} \cdot 2$

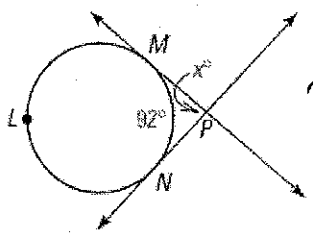
$144 = 200 - x$

$-200 \quad -200$

$-56 = -x$

$x = 56$

b.



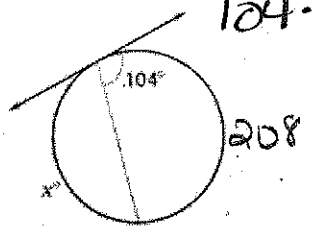
$360 - 92 = 268$

$\overset{\frown}{mLN} = 268$

$\frac{268 - 92}{2} = \frac{176}{2} = 88$

Write an equation that can be used to solve for x. Then solve the equation for x.

1.

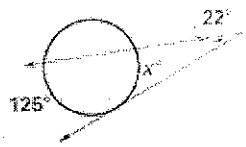


$104 \cdot 2 = 208$

$\frac{104 + 208 =}{2}$

$\frac{312}{2} = 152$

2.



$2 \cdot 22 = \frac{125 - x}{2} \cdot 2$

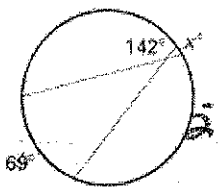
$44 = 125 - x$

$-125 \quad -125$

$-81 = -x$

$x = 81$

3.



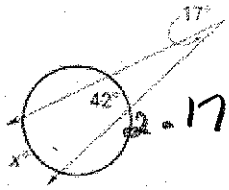
$38 = \frac{69 + x}{2} \cdot 2$

$76 = 69 + x$

$-69 \quad -69$

$x = 7$

4.



$2 \cdot 17 = \frac{x - 42}{2} \cdot 2$

$34 = x - 42$

$+42 \quad +42$

$76 = x$