# 7.2 Similar Polygons

Target: _Use proportions to identify similar polygons_
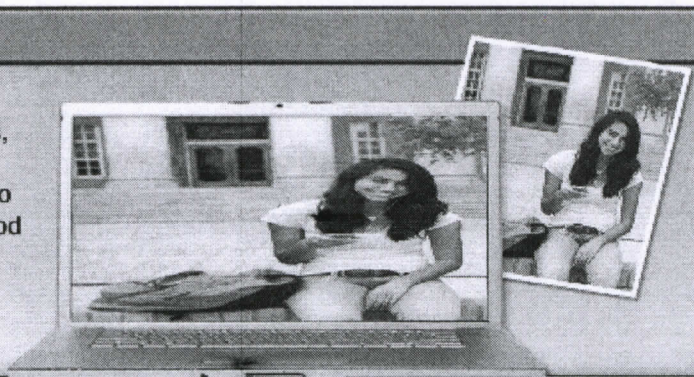
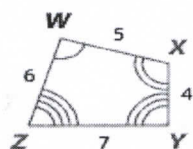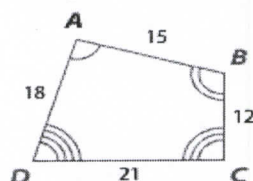## KeyConcept  Similar Polygons

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

**Example**  In the diagram below, *ABCD* is similar to *WXYZ*.

Corresponding angles

$\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, and $\angle D \cong \angle Z$

Corresponding sides

$$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{3}{1}$$

handwritten: $5 \div \frac{15}{5}$, $5 \div \frac{15}{5}$

**Symbols**  *ABCD ~ WXYZ*     ~ similar sign     $\frac{3}{1}$

As with congruence statements, the order of vertices in a similarity statement like *ABCD ~ WXYZ* is important. It identifies the corresponding angles and sides.
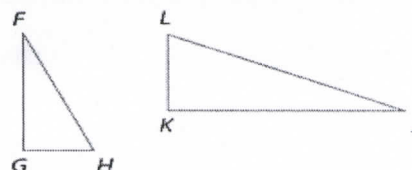
## Example 1  Use a Similarity Statement

If △*FGH ~* △*JKL*, list all pairs of congruent angles, and write a proportion that relates the corresponding sides.

$\angle F \cong \angle J$

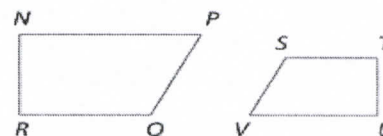$\angle G \cong \angle K$

$\angle H \cong \angle L$

Sides

$$\frac{FG}{JK} = \frac{GH}{KL} = \frac{FH}{JL}$$

▶ Guided Practice

1. In the diagram, *NPQR ~ UVST*. List all pairs of congruent angles, and write a proportion that relates the corresponding sides.

$\angle N \cong \angle U$

$\angle P \cong \angle V$

$\angle Q \cong \angle S$

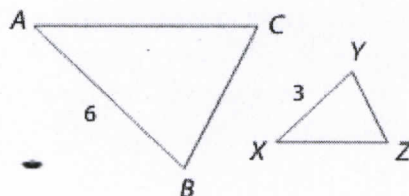$\angle R \cong \angle T$

Sides

$$\frac{NP}{UV} = \frac{PQ}{VS} = \frac{QR}{ST} = \frac{RN}{TU}$$

The ratio of the lengths of the corresponding sides of two similar polygons is called the **scale factor**. The scale factor depends on the order of comparison.

In the diagram, $\triangle ABC \sim \triangle XYZ$.

The scale factor of $\underline{\triangle ABC}$ to $\underline{\triangle XYZ}$ is $\frac{6}{3}$ or 2. $\frac{2}{1}$

The scale factor of $\underline{\triangle XYZ}$ to $\underline{\triangle ABC}$ is $\frac{3}{6}$ or $\frac{1}{2}$.



A ———— C
Y
6    3
X ———— Z
B

### Real-World Example 2  Identify Similar Polygons

**PHOTO EDITING** Kuma wants to use the rectangular photo shown as the background for her computer's desktop, but she needs to resize it. Determine whether the following rectangular images are similar. If so, write the similarity statement and scale factor. Explain your reasoning.
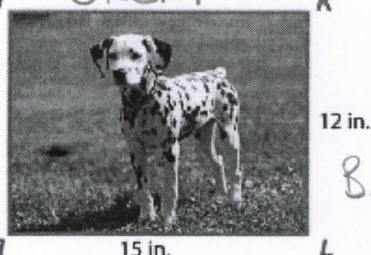
ABCD

A ————— B
8 in.
D   10 in.   C

**a.** E   EFGH   F

12 in.

H   14 in.   G

**b.** J   JKLM   K

12 in.

M   15 in.   L

A.
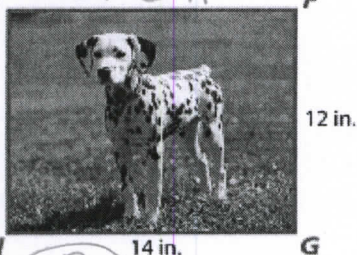
$$\frac{BC}{FG} = \frac{8 \div 4}{12 \div 4} = \boxed{\frac{2}{3}}$$  They are different

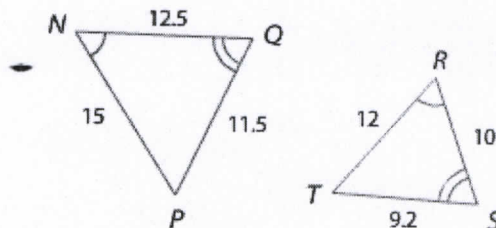$$\frac{CD}{GH} = \frac{10 \div 2}{14 \div 2} = \boxed{\frac{5}{7}}$$  so not similar

B. $$\frac{BC}{KL} = \frac{8 \div 4}{12 \div 4}\boxed{\frac{2}{3}}$$

$$\frac{CD}{LM} = \frac{10 \div 5}{15 \div 5}\boxed{\frac{2}{3}}$$

These are the same so they are similar.

**Guided Practice**

**2.** Determine whether the triangles shown are similar. If so, write the similarity statement and scale factor. Explain your reasoning.

N   12.5   Q
15    11.5
P

R
12    10
T   9.2   S

## 2 Use Similar Figures
**Use Similar Figures** You can use scale factors and proportions to solve problems involving similar figures.

Example 3 Use Similar Figures to Find Missing Measures

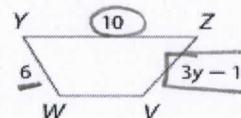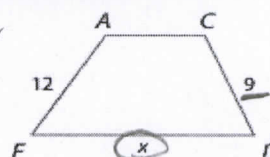In the diagram, $ACDF \sim VWYZ$.

**a.** Find $x$.

$$\frac{DF}{4Z} = \frac{x}{10}$$

$$\frac{CD}{W4} = \frac{9}{6}$$

$$\frac{x}{10} \diagup\!\!\!\!\diagdown \frac{9}{6}$$

$$\frac{6x}{6} = \frac{90}{6}$$

$$x = 15$$

**b.** Find $y$.

$$\frac{VZ}{AF} = \frac{3y-1}{12}$$

$$\frac{W4}{CD} = \frac{6}{9}$$

$$\frac{3y-1}{12} \diagup\!\!\!\!\diagdown \frac{6}{9}$$

$$27y + 9 = 72$$
$$\quad\ \ -9 \quad\ -9$$

$$\frac{27y}{27} = \frac{81}{27}$$

$$\boxed{y = 3}$$

(diagram: trapezoid $ACDF$ with $A$, $C$ top; sides $12$ and $9$; $F$, $x$, $D$ bottom. Below, trapezoid $VWYZ$ with $Y$, $10$, $Z$; $6$; $3y-1$; $W$, $V$ bottom.)

### GuidedPractice

Find the value of each variable if $\triangle JLM \sim \triangle QST$.

(diagram: $\triangle JLM$ with $J$ at top, side $4$ to $M$, $3y-2$ on left, $6x-3$ on right, $L$ at bottom. $\triangle QST$ with $S$ at top, $3$ left side, $5$ right side, $T$ left, $2$ bottom, $Q$ right.)

**3A.** $x$
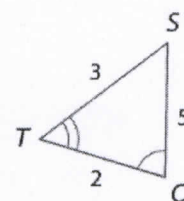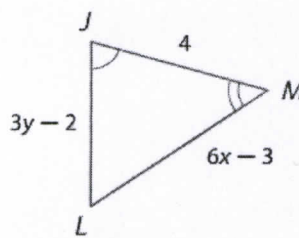
**3B.** $y$

In similar polygons, the ratio of any two corresponding lengths is proportional to the scale factor between them. This leads to the following theorem about the perimeters of two similar polygons.

## Theorem 7.1  Perimeters of Similar Polygons

If two polygons are similar, then their perimeters are proportional to the scale factor between them.

**Example** If $ABCD \sim JKLM$, then

$$\frac{AB + BC + CD + DA}{JK + KL + LM + MJ} = \frac{AB}{JK} = \frac{BC}{KL} = \frac{CD}{LM} = \frac{DA}{MJ}.$$
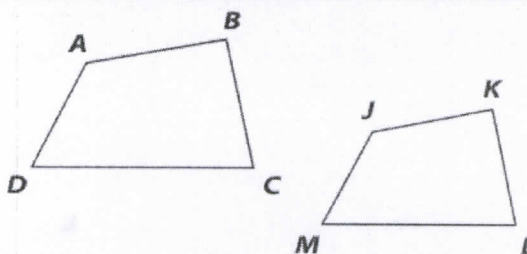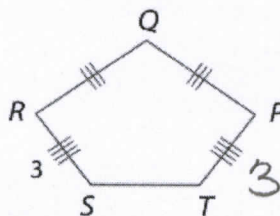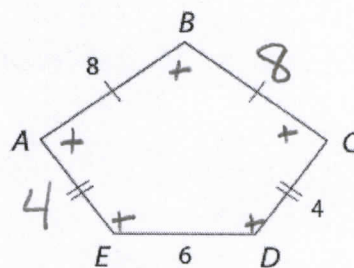
## Example 4  Use a Scale Factor to Find Perimeter

If $ABCDE \sim PQRST$, find the scale factor of $ABCDE$ to $PQRST$ and the perimeter of each polygon. Perimeter

$$\frac{AE}{PT} = \frac{4}{3} \times \frac{30}{X}$$

$$4x = \frac{90}{4}$$

$$X = 22.5$$

## GuidedPractice

4. If $MNPQ \sim XYZW$, find the scale factor of $MNPQ$ to $XYZW$ and the perimeter of each polygon.